

DYNAMICS OF SPACE TETHER SYSTEMS

by
Vladimir V. Beletsky
Evgenii M. Levin



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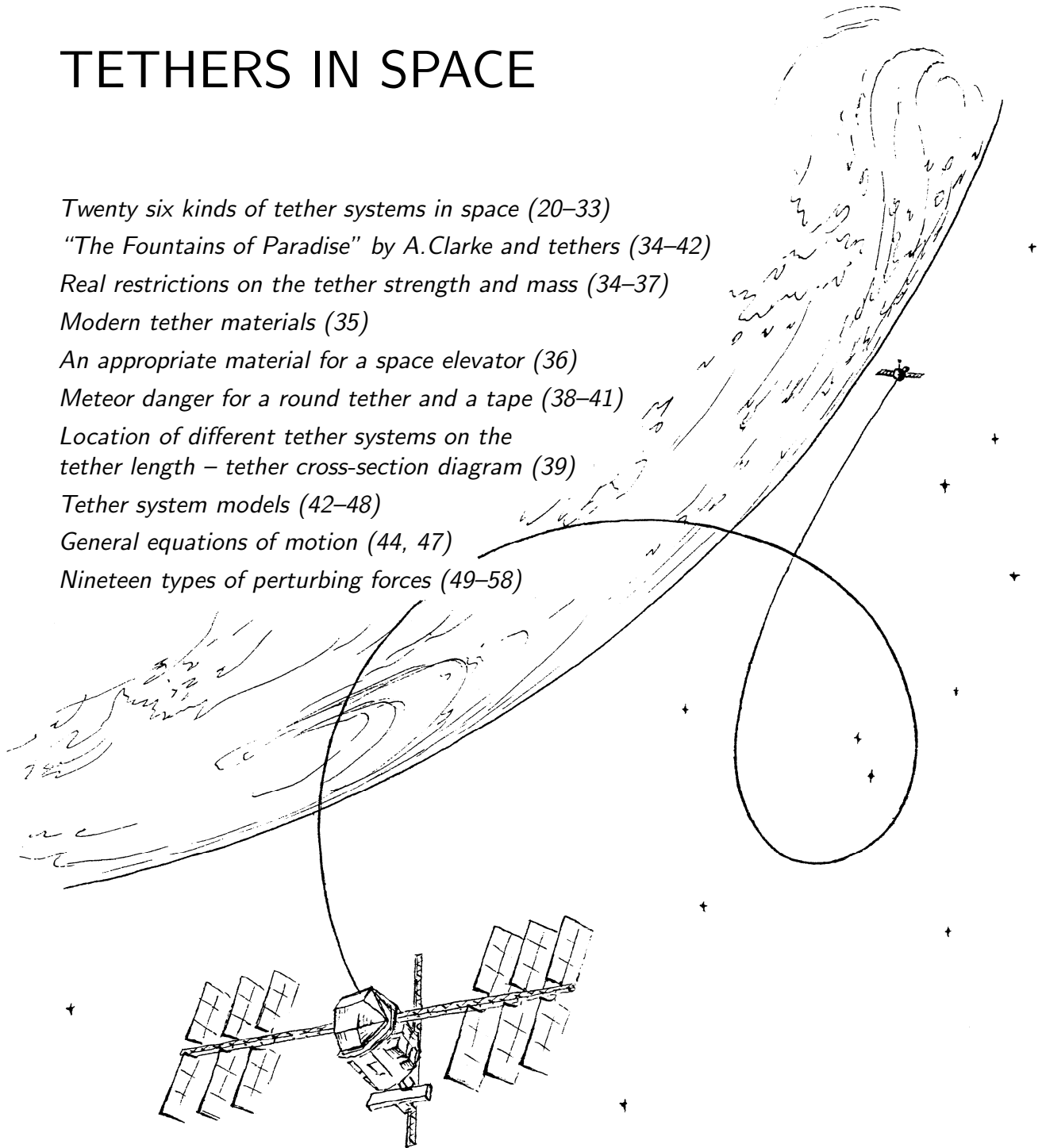
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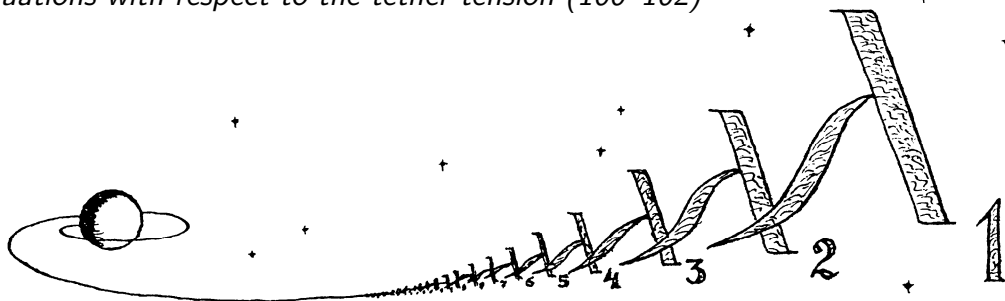
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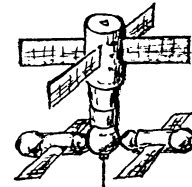
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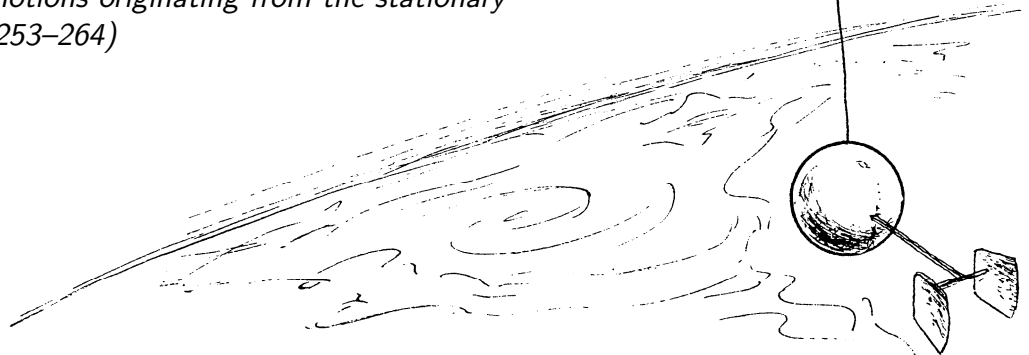
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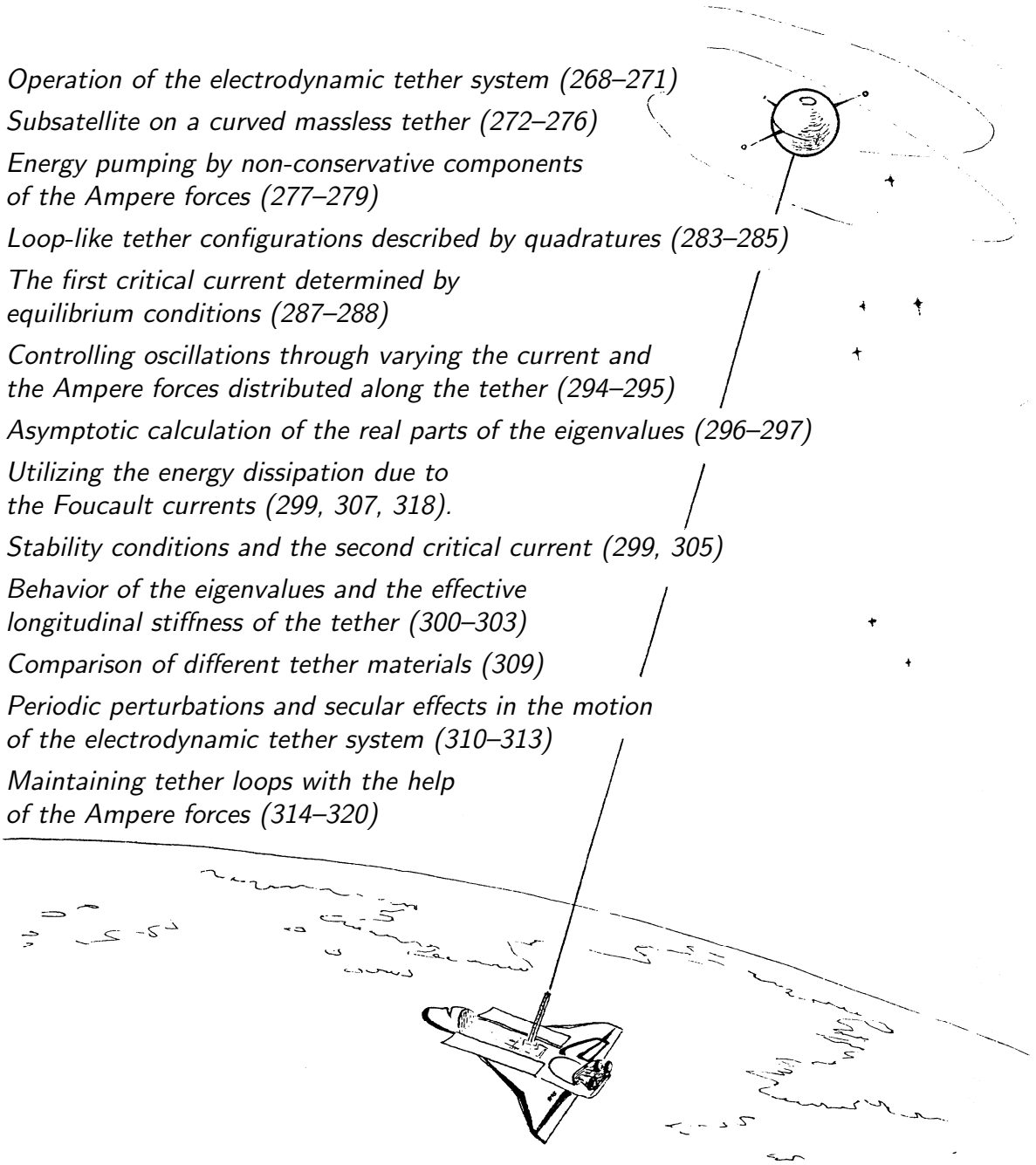
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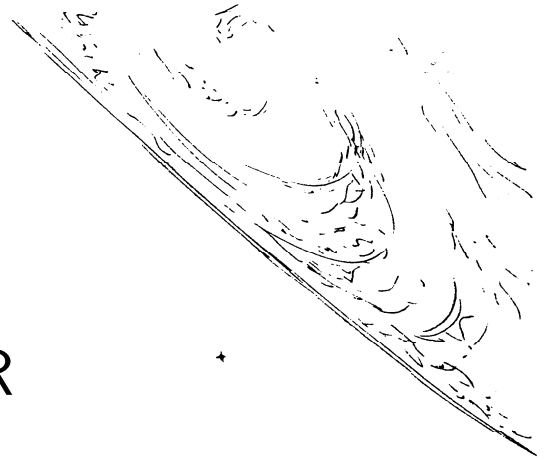
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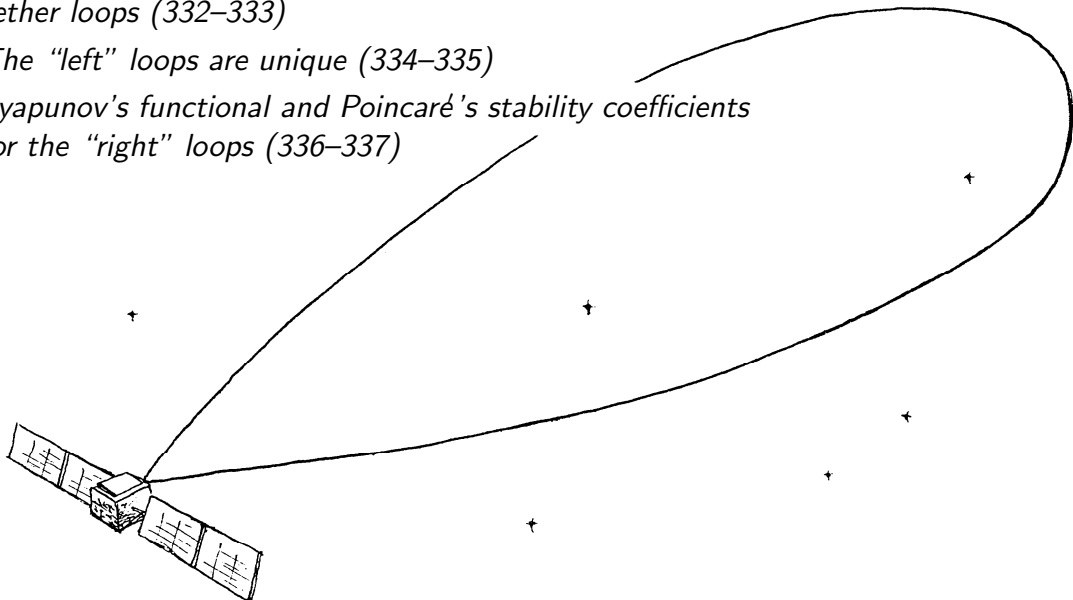
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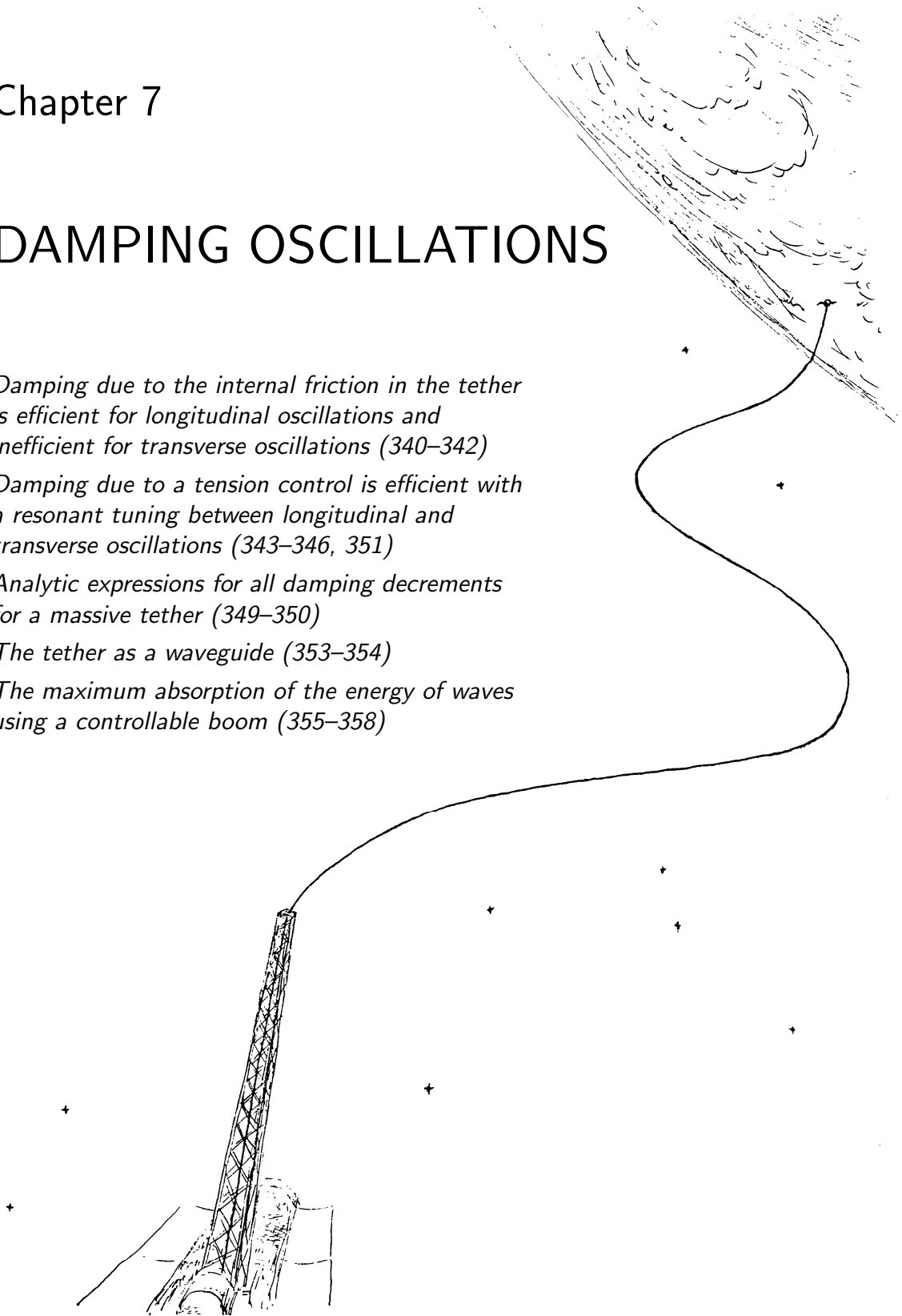
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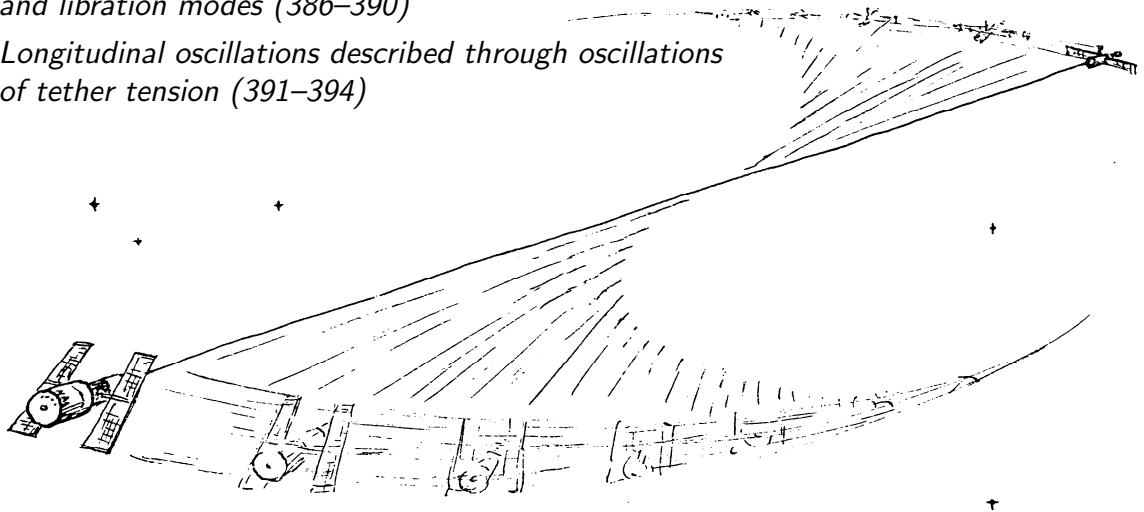
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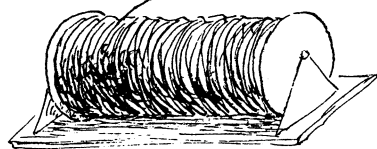
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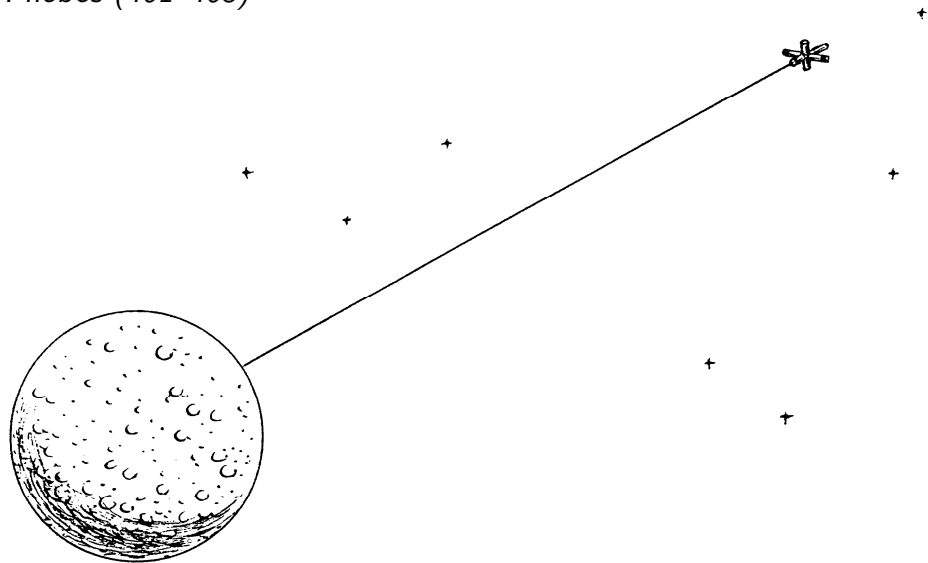
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