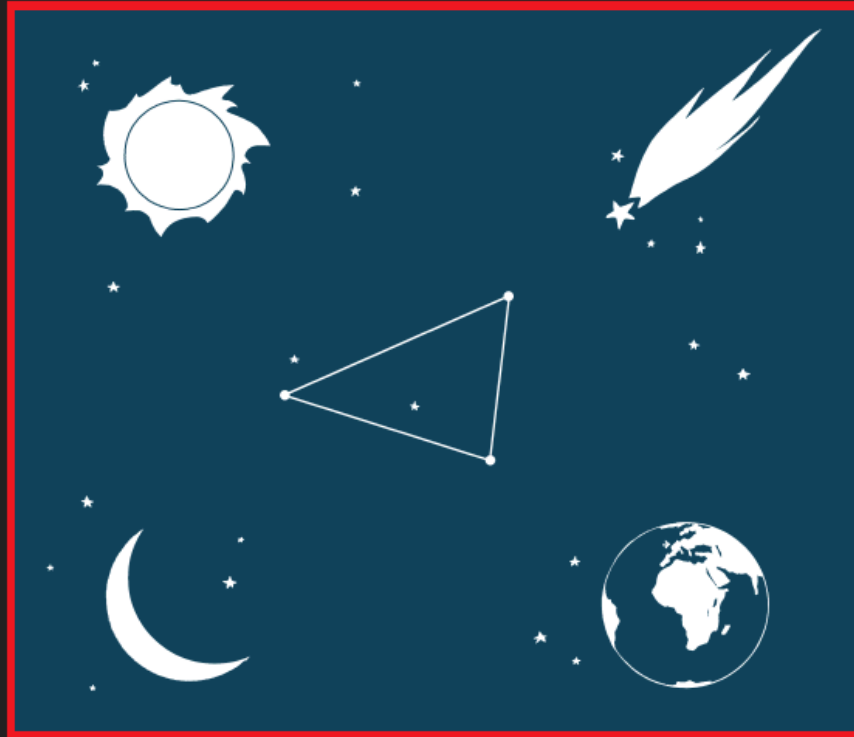


# DYNAMIC ANALYSIS OF SPACE TETHER MISSIONS

by  
Eugene M. Levin



Volume 126

ADVANCES IN THE ASTRONAUTICAL SCIENCES





# **DYNAMIC ANALYSIS OF SPACE TETHER MISSIONS**

**Volume 126  
ADVANCES IN THE ASTRONAUTICAL SCIENCES**

**by  
Eugene M. Levin**

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## FOREWORD

The American Astronautical Society is pleased to offer this text on the *Dynamic Analysis of Space Tether Missions* by Dr. Eugene M. Levin as a volume in the *Advances in the Astronautical Sciences* series. This book follows the first book on the *Dynamics of Space Tether Systems* by V. V. Beletsky and E. M. Levin published in the *Advances in the Astronautical Sciences* in 1993 as Volume 83. Previously, the Russian language edition of the book by Beletsky and Levin was published in 1990.

The AAS has had a continuing interest in the application of space tether technology since it had the honor to serve as a co-sponsor of the first NASA/PSN International Conference on Tethers in Space, September 17-19, 1986 in Arlington, Virginia, along with the AIAA and Associazione Italiana di Aeronautica e Astronautica (AIDAA). The proceedings of this conference were published by the AAS as Volume 62 of the *Advances in the Astronautical Sciences*. Subsequently, the AAS was also privileged to serve as a co-sponsor and participating organization for the Second and Third International Conferences on Tethers in Space.

Since 1993 Dr. Levin has participated in the dynamic analysis of several tether missions including comet nucleus sample return, reboost system for the Mir Space Station, electrodynamic autonomous transfer vehicles, momentum exchange tether system, deep space interferometer and others. Many of these studies produced unpredictable, surprising results and some were included only in contract reports and not in refereed archival publications. Some of these studies provide the basis for chapters in this new book.

As in the previous book this text is written in consistent notation throughout. The reader will appreciate the author's keen geometrical and mathematical insight, particularly in the area of partial differential equations. The book contains many very clear illustrations together with relevant practical numerical examples, some based on extensive numerical simulations. All chapters begin with a brief introduction describing the project within the context of advances in space technology during a particular time period. The projects discussed reflect the author's first-hand, direct experience. Dr. Levin cites 460 references, many describing very recent contract reports.

These references complement the 215 references contained in the earlier book. For all those who have an interest and experience in space-based tether systems and missions this book will represent a "must have" addition to their current tether systems library, a text they will often refer to in the future.

**Peter M. Bainum**  
**Distinguished Professor of**  
**Aerospace Engineering, Emeritus;**  
**Member, Board of Directors and**  
**International Programs Committee,**  
**American Astronautical Society**

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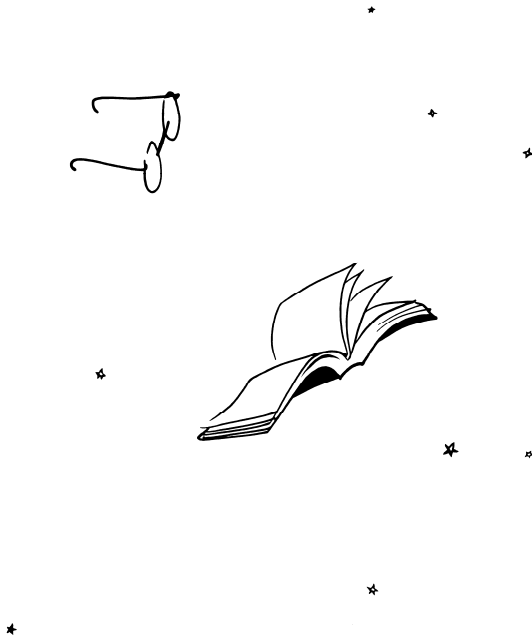
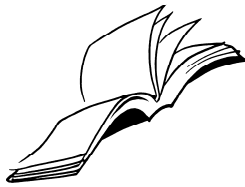
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## ABOUT THE BOOK

This book follows the first book on the “Dynamics of Space Tether Systems” by V.V. Beletsky and E.M. Levin published in the *Advances in the Astronautical Sciences* in 1993. Since that time, seven tether experiments have flown and nearly a thousand technical papers on various aspects of tether design and operation have been published. Space tether theory and technology have been advancing at an amazing pace.

During these years, the author had the privilege to be involved in the dynamic analysis of several tether missions, including comet nucleus sample return, Mir reboost system, electrodynamic autonomous orbit transfer vehicle, momentum exchange tether system, deep space interferometer, and others.

The results obtained in these studies were often surprising and not obvious even for a devoted tether dynamics veteran of twenty five years. There was much to be learned. In the process, the arsenal of the analytical tools of tether dynamics was widened and modified to solve particular problems. Some new tools were forged and some old tools were sharpened.

Based on these results, this book attempts to systematically describe effective approaches and modern techniques of the dynamic analysis of space tether systems in application to different missions with different operation scenarios and offer new insights into the dynamic behavior of space tethers.

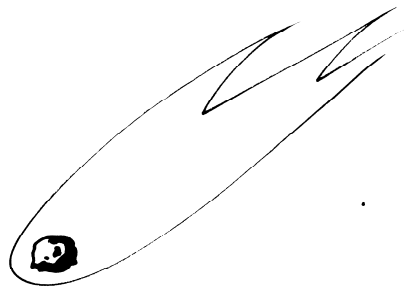
Each chapter of the book is devoted to a particular project in which the author had first-hand experience. All chapters start with a brief introduction that puts the project in context of the advances in space technology during the corresponding time period. Analysis begins with a basic dynamic model and equations of motion and proceeds with derivations that can be easily followed. It focuses on fundamental dynamic relations and phenomena, while trying to abstract from the many technical

details of a particular mission. Clear geometrical interpretations are always favored, and all analytical results are verified by extensive simulations.

The inspiration and significant effort that went into the preparation of the manuscript will be rewarded if the ideas and results discussed in this book will help better understand tether dynamics and create more efficient tools for dynamic analysis of space tether missions.

In the age of computerization, the author admits his old-fashioned appreciation of analytical methods and undoubtedly attributes it to the inspiring influence of Prof. V.V. Beletsky, one of the founders of the theory of attitude motion of satellites and one of the pioneers of space tether research.

The author would like to thank Prof. P.M. Bainum, J. Pearson, J.A. Carroll, and M.E. Levin for their helpful comments. Special thanks go to Irina Levin whose imaginative drawings provide a refreshing artistic introduction to each chapter.



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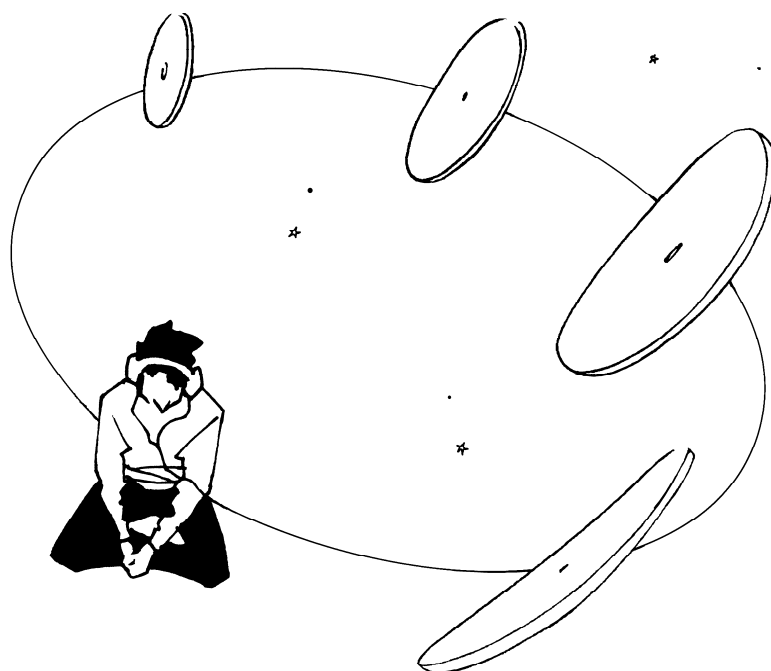
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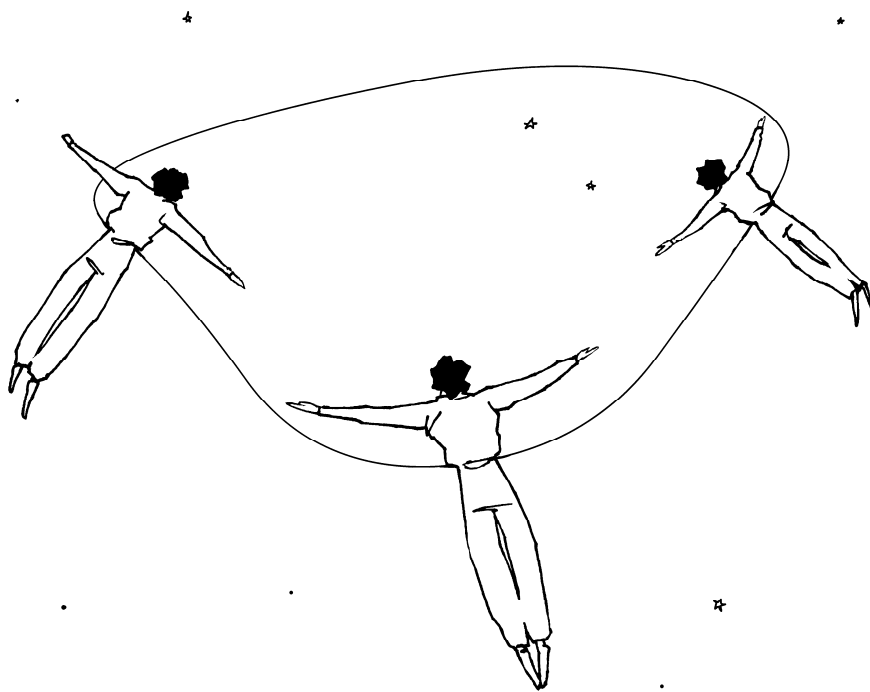
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